Models of Set Theory I – Summer 2017

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Solve the following problems, assuming ZFC to hold.

Problem 1 [4 points]:

- For which axioms φ of ZFC does φ^{Ord} hold?
- Let $X = \{y \mid y \subseteq Ord\}$. For which axioms φ of ZFC does φ^X hold?

Problem 2 [4 points]: Let $\{W_i \mid i < \omega\}$ be a collection of transitive sets such that for each axiom φ of ZFC and each $i < \omega$, φ^{W_i} holds.

- Show that the ZFC axioms of Pairing, Union and Infinity hold in $\bigcap_{i<\omega} W_i$, and that they hold in $\bigcup_{i<\omega} W_i$ in case the W_i form an increasing sequence, that is $W_i \subseteq W_j$ whenever $0 \leq i < j < \omega$.
- Assume that whenever $i < j$, then there is a subset of ω which is an element of $W_j \setminus W_i$. Show that under this assumption, $(Power)^{(\bigcup_{i<\omega} W_i)}$ fails to hold.

Problem 3 [4 points]:

- Assume that M is transitive and that $x \in M$. Show that

$$(x \in Ord)^M \iff x \in (Ord \cap M).$$

- Find a transitive set X such that $(Pairing)^X$ holds, however $(Union)^X$ fails.

Problem 4 [8 points]: Given a cardinal κ, we define

$$H_\kappa = \{x \mid card(TC(\{x\})) < \kappa\}.$$

Examine which ZFC axioms hold in H_κ for various infinite cardinals κ.